

حل تشریحی از سوال خارج از کشور ۹۸ رشته ریاضی

تعداد اعضای هر دو مجموعه = $|A| = 10$

(۱.۱) گزینه ۴

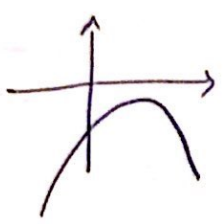
تعداد اعضای هر دو مجموعه = $|F| = 12 \Rightarrow |A \cup F| = |A| + |F| - |A \cap F|$
 تعداد اعضای هر دو مجموعه = $|A \cap F| = 5 \Rightarrow 10 + 12 - 5 = 22$

$\Rightarrow |A' \cap F'| = 52 - 22 = 30$

(۱.۲) گزینه ۳

$$A = \sqrt{\sqrt{3^5}} \times \left(\frac{1}{12}\right)^{\frac{2}{3}} = \sqrt{3^5} \times \left(\frac{1}{12}\right)^{\frac{2}{3}} = \frac{3^{\frac{5}{2}}}{(2 \times 3)^{\frac{2}{3}}} = \frac{1}{2^{\frac{2}{3}} \times 3^{\frac{1}{3}}} = \frac{1}{2}$$

 $\Rightarrow \left(1 + \frac{1}{A}\right)^{\frac{1}{2}} = (2.5)^{\frac{1}{2}} = \sqrt{2.5} = 5$

(۱.۳) گزینه ۲


$$\Delta < 0 \Rightarrow 4(m-2)^2 + 4(1-m) < 0 \Rightarrow m^2 - 4m + 1 < 0$$

$$\Delta < 0 \Rightarrow 1-m < 0 \Rightarrow \frac{m > 1}{0}$$

$$(m-2)(m-1) < 0$$

$$\frac{2 < m < 1}{0}$$

$$A \cap B = \{2 < m < 1\}$$

(۱.۴) گزینه ۴

$$f(x+2) - 9 = (x+2)^2 - (x+2) - 9 = x^2 + 4x - 1 < 0 \Rightarrow (x-5)(x+2) < 0$$

 $\Rightarrow -2 < x < 5$

(۱.۵) گزینه ۱

$$A = \frac{1}{2} \left(\frac{5-2}{2 \times 5} + \frac{1-5}{5 \times 1} + \frac{11-1}{1 \times 11} + \dots + \frac{20-17}{17 \times 20} \right)$$

$$A = \frac{1}{2} \left(\frac{1}{5} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{11} + \dots + \frac{1}{17} - \frac{1}{20} \right)$$

$$A = \frac{1}{2} \left(\frac{1}{5} - \frac{1}{20} \right) = \frac{1}{2} \left(\frac{4}{20} \right) = \frac{1}{10} = 0.1$$

(۱.۶) گزینه ۲

$$m < -2 \rightarrow 1 - 2m - m - 2 = 3 \Rightarrow 3m = -5 \Rightarrow m = -\frac{5}{3}$$

$$-2 < m < \frac{1}{2} \rightarrow 1 - 2m + m + 2 = 3 \Rightarrow m = 0$$

$$m > \frac{1}{2} \rightarrow 2m - 1 + m + 2 = 5 \Rightarrow 3m = 4 \Rightarrow m = \frac{4}{3}$$

$$g^{-1} = \{(0, 2), (2, 0), (4, 2), (1, 0)\} \Rightarrow g^{-1} \circ f = \{(1, 0), (0, 2)\}$$

$$\Rightarrow (g^{-1} \circ f) - f = \{(1, 2), (0, -1)\} \Rightarrow r = \{-1, 2\}$$

$$n | 1 \in f \rightarrow f(1) = 1 \Rightarrow e^{A+B} = 1 = e^0 \Rightarrow A+B=0$$

$$n | 9 \in f \rightarrow f(9) = 9 \Rightarrow e^{9A+B} = 9 = e^2 \Rightarrow 9A+B=2$$

$$\Rightarrow f(x) = 3^{x-1} \rightarrow f(1) = 3^{-1} = \frac{1}{3}$$

$$A = \tan(\pi - \frac{\pi}{4}) \cdot \sin(\pi - \frac{\pi}{4}) + \cos(\pi - \frac{\pi}{4}) = \tan(-\frac{\pi}{4}) \cdot \sin(-\frac{\pi}{4}) + \cos \frac{\pi}{4}$$

$$= (-\frac{\sqrt{e}}{e})(-\frac{\sqrt{e}}{e}) + (-\frac{1}{e}) = 0$$

$$\lim_{x \rightarrow 1^+} \frac{\sin^2 \pi x}{1 + \cos \pi x} = \lim_{x \rightarrow 1^+} \frac{1 - \cos^2 \pi x}{1 + \cos \pi x} = \lim_{x \rightarrow 1^+} 1 - \cos \pi x = 1 - (-1) = 2$$

مجموعه دامنه $f \Rightarrow -1, 1, \dots$

$$f(x) = \begin{cases} x[x]; & -1 < x < 1 \\ ax+b; & x \geq 1, x \leq -1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = a+b \rightarrow a+b=0$$

$$\lim_{x \rightarrow -1^+} f(x) = -a+b \quad \lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} -x = 1 \Rightarrow b-a=1$$

$$\mathbb{R}, \mathbb{R} \Rightarrow 2b=1 \Rightarrow b=\frac{1}{2}, a=-\frac{1}{2}$$

$$f(x) = \tan \pi x - \cot \pi x = -2 \cot 2\pi x \rightarrow T = \frac{\pi}{2\pi} = \frac{1}{2}$$

$$1 - 2 \sin^2 \pi x = \frac{1}{2} \rightarrow \sin^2 \pi x = \frac{1}{4} \Rightarrow \frac{1}{2} \sin^2 2\pi x = \frac{1}{4}$$

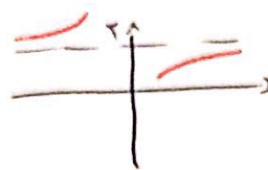
$$\rightarrow \sin^2 2\pi x = 1 \rightarrow \sin 2\pi x = 1 \rightarrow 2m = k\pi + \frac{\pi}{2} \rightarrow n = k\pi + \frac{\pi}{2}$$

$$\rightarrow \sin 2\pi x = -1 \rightarrow 2m = k\pi - \frac{\pi}{2} \rightarrow n = k\pi - \frac{\pi}{2}$$

$$\rightarrow n = k\pi \pm \frac{\pi}{2}$$

$\lim_{x \rightarrow \infty} y = r \rightarrow y = r$

$\lim_{x \rightarrow +\infty} (y-r) = \lim_{x \rightarrow +\infty} \frac{-x-r}{x^2+2x} = 0 \text{ (0)}$



$\lim_{x \rightarrow -\infty} (y-r) = \lim_{x \rightarrow -\infty} \frac{-x-r}{x^2+2x} = 0 \text{ (0)}$

$y = cx - d \rightarrow y = 1 \rightarrow |g| \rightarrow g(x) = 1$

$m_{\text{at}} = r \rightarrow g(x) = r \quad \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \frac{1}{c} f'(1) = \frac{r}{c} \rightarrow f'(1) = \frac{rc}{1}$

$(f \circ g)'(x) = g'(x) \cdot f'(g(x)) = c f'(1) = c \left(\frac{rc}{1}\right) = c^2$

$D_f: \mathbb{R} - \{0\} \quad f(x) = \ln|x^2 - 2|$

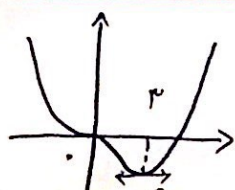
$x = \pm\sqrt{2}$ ساده راضی از کجا، و نکته \sqrt{x} (فوش زار و بار) و متن f در \sqrt{x} تمام توابع نشده این نکته، نکته \sqrt{x} متن f و \sqrt{x} متن f

$\frac{f(x) - f(0)}{x - 0} = \frac{e + \frac{1}{e} - 2}{e} = \frac{e}{1}$

$[0, e]$

$f'(x) = \frac{1}{\sqrt{x+1}} \cdot \frac{1}{(x+1)^2} \Big|_{x=e} = \frac{1}{e} - \frac{e}{10} = \frac{10}{e}$

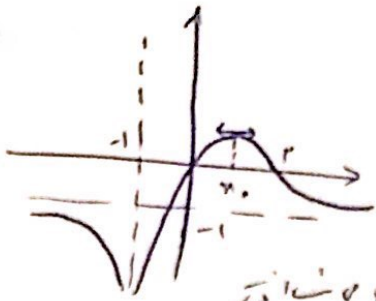
$\frac{10}{e} - \frac{e}{10} = \frac{2}{e} = 1/e$



$f(x) = nx^2 + ax^2 + bx^2$
 $f(0) = 0 \rightarrow f'(0) = 0 \rightarrow b = 0$
 $f''(0) = 0$

$f(x) = 14 + 4x = 14$

$f(x) = 0 \rightarrow nx^2 + ax^2 = 0 \Rightarrow 14 + 4x = 0 \Rightarrow a = -14$



$$f'(x) = \frac{(1-x)(n+1)^x - x(n+1)^{x-1}(1-x)}{(n+1)^x} = \frac{x(1-x) - x(n+1)^{-1}(1-x)}{(n+1)^x} = \dots$$

$$\Rightarrow x = \frac{1}{c} \text{ (critical point)} \rightarrow \text{max} \left| \frac{1}{c} \right|$$

$$\text{Interval } = \frac{1}{c} - (-1) = \left[-1, \frac{1}{c} \right]$$

$$D_f \in [-c, c] - \{1\}$$

$$\rightarrow k-r \geq -c \rightarrow k \geq -1$$

$$c k + r \leq c \rightarrow k \leq \frac{1}{c} \rightarrow \left[-1, \frac{1}{c} \right)$$

$$k = \frac{1}{c}$$

$$f = \dots (12)$$