

پاسخ تشریحی سرکاری تجربی ۹۹ داخل کثور

دور ریاضی - حین حاجیلو

(۲) - ۱۲۶

$$\frac{\sqrt{8} + \sqrt{27}}{5 - \sqrt{6}} = \frac{2\sqrt{2} + 3\sqrt{3}}{5 - \sqrt{6}} \times \frac{5 + \sqrt{6}}{5 + \sqrt{6}} = \frac{10\sqrt{2} + 15\sqrt{2} + 4\sqrt{3} + 9\sqrt{2}}{25 - 6} = \frac{19\sqrt{2} + 19\sqrt{3}}{19}$$

$$2(\sqrt{9} - 1)^{-1} = \frac{2}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{2(\sqrt{3} + 1)}{2} = \sqrt{3} + 1 = \sqrt{2} + \sqrt{3}$$

$$\Rightarrow (\sqrt{2} + \sqrt{3}) - (\sqrt{3} + 1) = \sqrt{2} - 1$$

① {1}

② {2, 3, 4}

③ {5, 6, 7, 8, 9}

⋮

④ { $8^2 + 1, \dots, 9^2$ } $\Rightarrow \frac{(8^2 + 1) + 9^2}{2} = 73$

(۳) - ۱۲۷

$$p(x) = (x^2 - 1) \cdot A(x) \Rightarrow P(1) = P(-1) = 0$$

(۲) - ۱۲۸

$$Q(x) = p(x-1) + p(1-x) \Rightarrow \text{بابتی نده تقسیم بر } x-2 = Q(2) = p(1) + p(-1) = 0$$

$$\delta = \frac{1}{p} \Rightarrow \frac{-b}{a} = \frac{a}{c} \Rightarrow a^r = -bc$$

① - 129

$$\Rightarrow q = (r^{m-1})(m-r) \Rightarrow q = r^{m-1} - \omega m + r \Rightarrow r^{m-1} - \omega m - r = 0$$

$$\Rightarrow \begin{cases} m = -1 \rightsquigarrow \Delta < 0 \\ m = \frac{r}{r} \quad \checkmark \end{cases}$$

$$\begin{aligned} & \frac{r(x+1) - r}{x+1} = r - \frac{r}{x+1} \\ 1 < \frac{x+1}{rx-1} < r & \Rightarrow \frac{1}{r} < \frac{rx-1}{x+1} < 1 \Rightarrow -1 < \frac{r}{x+1} - r < -\frac{1}{r} \\ \Rightarrow 1 < \frac{r}{x+1} < \frac{\omega}{r} & \Rightarrow \frac{r}{\omega} < \frac{x+1}{r} < 1 \Rightarrow \frac{r}{\omega} < x+1 < r \\ \Rightarrow \frac{r}{\omega} < x < r & \quad \checkmark \end{aligned}$$

② - 130

$$y = ax^r + bx + c \xrightarrow{(0, \omega)} c = \omega \Rightarrow y = ax^r + bx + \omega$$

③ - 131

$$\begin{aligned} (1, 1) & \rightarrow 1 = a + b + \omega \Rightarrow a + b = \xi \\ (-r, \omega) & \rightarrow \omega = ra - rb + \omega \Rightarrow ra - b = 0 \end{aligned} \xrightarrow{\text{حل}} \begin{cases} a = r \\ b = \xi \end{cases}$$

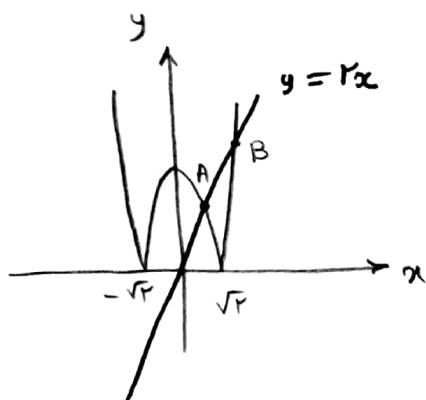
$$\Rightarrow y = rx^r + \xi x + \omega \rightsquigarrow (-1, r) \quad \checkmark$$

$$f(x) = \sqrt{x} \quad \rightarrow \quad f_1(x) = \sqrt{x-12} \quad \rightarrow \quad f_p(x) = r + \sqrt{x-12}$$

$$\begin{cases} y = \sqrt{x} \\ y = r + \sqrt{x-12} \end{cases} \Rightarrow \sqrt{x} = r + \sqrt{x-12} \Rightarrow x = r^2 + (x-12) + \varepsilon \sqrt{x-12}$$

$$\Rightarrow \varepsilon \sqrt{x-12} = \Lambda \Rightarrow \sqrt{x-12} = r \Rightarrow x-12 = \varepsilon \Rightarrow x = 14$$

$$\Rightarrow y = r \Rightarrow A(14, r) \Rightarrow OA = \sqrt{14^2 + r^2} = \sqrt{r^2(r^2+1)} = \varepsilon \sqrt{14}$$



$$A: \begin{cases} y = rx \\ y = r^2 x^2 \end{cases} \Rightarrow \varepsilon - r^2 x^2 = rx$$

$$\Rightarrow r^2 x^2 + rx - \varepsilon = 0 \Rightarrow x^2 + x - r = 0$$

$$\Rightarrow x = 1$$

$$B: \begin{cases} y = rx \\ y = r^2 x^2 - \varepsilon \end{cases} \Rightarrow r^2 x^2 - \varepsilon = rx \Rightarrow r^2 x^2 - rx - \varepsilon = 0$$

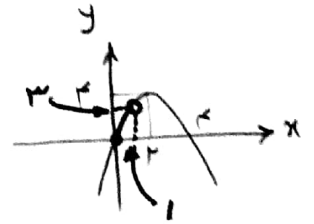
$$\Rightarrow x^2 - x - r = 0 \Rightarrow x = r$$

$$\Rightarrow \text{Max}(b-a) = r-1 = 1$$

① - 13ε

$$f(x) = rx - [rx] \Rightarrow 0 \leq f(x) < 1$$

$$g(x) = -x^r + \varepsilon x \Rightarrow g(x) = -(x-r)^r + \varepsilon$$



$$0 \leq x < 1 \Rightarrow 0 \leq g(x) < \mu$$

② - 13ε

$$f^{-1}(9) = a \Rightarrow f(a) = 9 \Rightarrow a + \sqrt{a} = 9 \Rightarrow a = 8$$

$$f^{-1}(12) = b \Rightarrow f(b) = 12 \Rightarrow b + \sqrt{b} = 12 \Rightarrow b = 9$$

$$\Rightarrow a + b = 17$$

③ - 13ε

$$f: y = x - \frac{r}{x} \Rightarrow f^{-1}: x = y - \frac{r}{y} \xrightarrow{y = -x}$$

$$\Rightarrow -y = y - \frac{r}{y} \Rightarrow \frac{r}{y} = 2y \Rightarrow y^2 = 1 \Rightarrow y = \pm 1 \xrightarrow{y < 0}$$

$$y = -1 \xrightarrow{x = -y} x = 1$$

① - 13V

$$\log_r r = 1 \Rightarrow \frac{1}{r} \log_r r = \frac{1}{r} \Rightarrow \log_r r = 1/r$$

$$\begin{aligned} \log_{1/r} 4 &= \frac{\log_r 4}{\log_r 1/r} = \frac{\log_r 4}{\log_r r^{-1}} = \frac{\log_r 4}{-\log_r r} = \frac{\log_r 4}{-1} = -\log_r 4 \\ &= -\frac{1 + 1/r}{r + 1/r} = -\frac{r + 1}{r^2 + 1} = -\frac{r^2 + 1}{r^2 + 1} = -1 \end{aligned}$$

$$f(x) = -x + r^{ax+b}$$

② - 13A

$$f\left(-\frac{1}{r}\right) = 0 \Rightarrow -x + r^{ax+b} = 0 \Rightarrow r^{ax+b} = x$$

$$\Rightarrow -\frac{1}{r} + 1 = r \Rightarrow a = -r$$

$$f(0) = -r \Rightarrow -r + r^b = -r \Rightarrow r^b = r^1 \Rightarrow b = 1$$

$$\Rightarrow f(x) = -x + r^{-rx+1}$$

$$\Rightarrow f\left(-\frac{1}{r}\right) = -x + r^y = 0$$

$$f^{-1}(r) = a \Rightarrow f(a) = r \Rightarrow \frac{r^a + r^{-a}}{r} = r$$

③ - 139

$$\Rightarrow r^a + r^{-a} = r \xrightarrow{r^a = t} t + \frac{1}{t} = r \Rightarrow t^2 - rt + 1 = 0$$

$$\Rightarrow t = \frac{r \pm \sqrt{r^2 - 4}}{2} = r \pm \sqrt{r} \xrightarrow{a > 0} r^a = r + \sqrt{r}$$

$$\Rightarrow a = \log_r (r + \sqrt{r})$$

$$\tan 140^\circ = \tan(180^\circ - 40^\circ) = -\tan 40^\circ = -\sqrt{3}$$

Ⓐ - 140

$$\cos 110^\circ = \cos(180^\circ - 70^\circ) = -\cos 70^\circ = -\frac{\sqrt{3}}{2}$$

$$\begin{aligned} \tan 190^\circ &= \tan(180^\circ + 10^\circ) = \tan 10^\circ = \tan(180^\circ - 170^\circ) \\ &= -\tan 170^\circ = -\sqrt{3} \end{aligned}$$

$$\begin{aligned} \sin 140^\circ &= \sin(180^\circ - 40^\circ) = \sin 40^\circ = \frac{\sqrt{3}}{2} \\ \sin 110^\circ &= \sin(180^\circ - 70^\circ) = \sin 70^\circ = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\Rightarrow (-\sqrt{3}) \left(-\frac{\sqrt{3}}{2}\right) + (-\sqrt{3}) \left(\frac{\sqrt{3}}{2}\right) = \frac{3}{2} - \frac{3}{2} = 0$$

$$y = a + b \sin\left(\frac{\pi}{3} + x\right) \Rightarrow y = a + b \cos x$$

Ⓐ - 141

بالاتر به مقدار $b < 0$

$$\max = a - b = 3 \quad \text{Ⓐ}$$

$$a + b \cos \frac{\pi}{3} = 0 \Rightarrow a + b \cos \frac{\pi}{3} = 0 \Rightarrow a + \frac{b}{2} = 0 \quad \text{Ⓑ}$$

$$\text{Ⓐ}, \text{Ⓑ} \Rightarrow a = 1, b = -2$$

بالاتر به مقدار a, b هم عددت نیستند.

$$\frac{2\pi}{|b|} = \frac{4\pi}{2} + \frac{2\pi}{2} = 4\pi \Rightarrow |b| = \frac{1}{3} \xrightarrow{b < 0} b = -\frac{1}{3} \quad \text{Ⓐ - 142}$$

$$\begin{cases} \max = 1 \\ \min = -3 \end{cases} \xrightarrow{a > 0} \begin{cases} a + c = 1 \\ -a + c = -3 \end{cases} \xrightarrow{\text{حذف } a} \begin{cases} a = 2 \\ c = -1 \end{cases}$$

$$\Rightarrow \frac{a}{b} = \frac{2}{-\frac{1}{3}} = -6$$

$$\sin\left(2x - \frac{\pi}{2}\right) = \cos\left(x + \frac{\pi}{2}\right) \Rightarrow \sin\left(2x - \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - \left(x + \frac{\pi}{2}\right)\right)$$

$$\frac{\pi}{2} - x$$

$$\Rightarrow \text{بکلی از جوابی: } 2x - \frac{\pi}{2} = 2k\pi + \frac{\pi}{2} - x \Rightarrow 3x = 2k\pi + \frac{\pi}{2} \Rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6}$$

$$\lim_{x \rightarrow -r^-} \frac{[x] + r}{x + r} = \frac{\overbrace{[-r^-]}^{-r} + r}{0^-} = \frac{\text{منفی صغیر}}{\text{منفی صغیر}} = 0 \quad \text{Ⓜ} - 144$$

$$\lim_{x \rightarrow \infty} \frac{ax - \sqrt{x^2 - 1}}{rx^n - 1r} = \frac{1}{9} \Rightarrow \begin{cases} n = 1 \\ \frac{a}{r} = \frac{1}{9} \Rightarrow a = \frac{r}{9} \end{cases} \quad \text{Ⓛ} - 145$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{\frac{r}{9}x - \sqrt{x^2 - 1}}{rx - 1r} \stackrel{\text{جوابی}}{=} \lim_{x \rightarrow \infty} \frac{\frac{r}{9} - \frac{rx}{r\sqrt{(x^2 - 1)^2}}}{r}$$

$$= \frac{\frac{r}{9} - \frac{1}{r}}{r} = \frac{\frac{1}{9}}{r} = \frac{1}{9r}$$

$$f(x) = \begin{cases} \sqrt{a - rx} & ; x \leq -r \Rightarrow f(-r) = \sqrt{a} = r \\ -\frac{1}{r}x^2 + bx + c & ; x > -r \Rightarrow \lim_{x \rightarrow -r^+} f(x) = -r - rb + c \end{cases} \quad \text{Ⓝ} - 146$$

$$\Rightarrow -r - rb + c = r \Rightarrow c - rb = a \quad (*)$$

$$f'(x) = \begin{cases} \frac{-r}{2\sqrt{a - rx}} & ; x < -r \Rightarrow f'_-(-r) = -\frac{1}{r} \\ -x + b & ; x > -r \Rightarrow f'_+(-r) = r + b \end{cases}$$

$$\Rightarrow r + b = -\frac{1}{r} \Rightarrow b = -\frac{1}{r} \xrightarrow{(*)} c = \frac{1}{r}$$

② -14V

$$f(x) = \left(\frac{\sqrt{x^2 + 2x}}{x^2 - x} \right)^2 = \frac{x^2 + 2x}{(x^2 - x)^2} \Rightarrow$$

$$f'(x) = \frac{(2x+2)(x^2-x)^2 - 2(x^2-x)^1(2x-1)(x^2+2x)}{(x^2-x)^4}$$

$$\Rightarrow f'(2) = \frac{9 \times 4 - 2 \times 2 \times 2 \times 2 \times 4}{9 \times 4} = -\frac{10}{2}$$

① -14A

$$f(x) = x + \sqrt{\varepsilon x - x^2} \Rightarrow f'(x) = 1 + \frac{\varepsilon - 2x}{2\sqrt{\varepsilon x - x^2}} = \frac{2\sqrt{\varepsilon x - x^2} + (\varepsilon - 2x)}{2\sqrt{\varepsilon x - x^2}}$$

$$f'(x) = 0 \Rightarrow 2x - \varepsilon = 2\sqrt{\varepsilon x - x^2} \Rightarrow x - \frac{\varepsilon}{2} = \sqrt{\varepsilon x - x^2}$$

$$\Rightarrow x^2 - \varepsilon x + \varepsilon = \varepsilon x - x^2 \Rightarrow 2x^2 - 2\varepsilon x + \varepsilon = 0$$

$$\Rightarrow x^2 - \varepsilon x + \frac{\varepsilon}{2} = 0 \Rightarrow x = \frac{\varepsilon}{2} \pm \sqrt{\frac{\varepsilon}{2}}$$

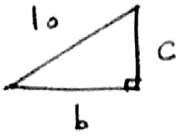
$$f(x) = x + \sqrt{\varepsilon - (x - \frac{\varepsilon}{2})^2} \begin{matrix} \xrightarrow{x = \frac{\varepsilon}{2} + \sqrt{\frac{\varepsilon}{2}}} y = \frac{\varepsilon}{2} + 2\sqrt{\frac{\varepsilon}{2}} \\ \xrightarrow{x = \frac{\varepsilon}{2} - \sqrt{\frac{\varepsilon}{2}}} y = \frac{\varepsilon}{2} \end{matrix}$$

فاصله هر کدام از نقاط $(\frac{\varepsilon}{2} - \sqrt{\frac{\varepsilon}{2}}, \frac{\varepsilon}{2})$ و $(\frac{\varepsilon}{2} + \sqrt{\frac{\varepsilon}{2}}, \frac{\varepsilon}{2} + 2\sqrt{\frac{\varepsilon}{2}})$ خط $y-x=0$

برابر با یک است :

$$\frac{|\frac{\varepsilon}{2} - (\frac{\varepsilon}{2} - \sqrt{\frac{\varepsilon}{2}})|}{\sqrt{1^2 + (-1)^2}} = 1$$

(۴) - ۱۴۹



$$V = \frac{1}{3} \pi b^2 c$$

$$= \frac{1}{3} \pi (100 - c^2) c$$

$$= \frac{1}{3} \pi (100c - c^3)$$

$$b^2 + c^2 = 10^2$$

$$b^2 = 100 - c^2 \quad (*)$$

$$\Rightarrow V' = \frac{1}{3} \pi (100 - 3c^2) = 0 \Rightarrow c = \frac{10}{\sqrt{3}}$$

(*) ↓

$$b = \frac{10\sqrt{2}}{\sqrt{3}}$$

(۳) - ۱۵۰

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 5 \end{pmatrix} = 2 \times 35 + 25 = 95$$

و یکی از دونو

همچنین از دونو

(۲) - ۱۵۱

$$P = \frac{2! \times 5! \times 3!}{8!} = \frac{1}{128}$$

Ⓟ - 152

$y = x - 10$	○ ○ ○ ○ ○ ♣ ♣ ♣ ♣ ♣ ♣ ♣ ♣ ♠ ♠ ♠ ♠ ♠ ♠ ♠ ♠
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$$\bar{y} = \frac{14}{14} = 1 \Rightarrow \bar{x} = 11$$

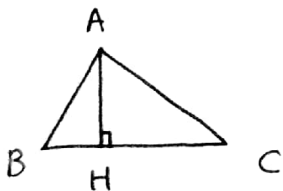
$$\sigma_y^2 = \frac{4 \times 1 + 1 \times 1 + 1 \times 1}{14} = \frac{6}{14} = \frac{3}{7} \Rightarrow \sigma_y = \sigma_x = \frac{\sqrt{14}}{7}$$

$$CV_x = \frac{\sigma_x}{\bar{x}} = \frac{\sqrt{14}}{11} \approx 0.12$$

Ⓟ - 182

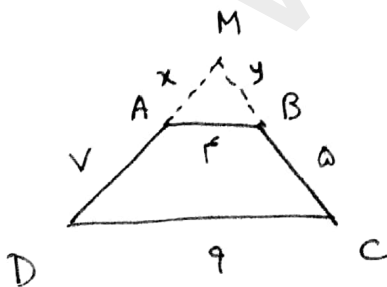
$$BC: y + 1 = \frac{1+1}{1-1} (x-1) \Rightarrow y = x - 1$$

$$y - x + 1 = 0$$



$$\Rightarrow AH = \frac{|1 - 1 + 1|}{\sqrt{1}} = \frac{1}{\sqrt{1}} = 1 = \varepsilon \sqrt{1}$$

Ⓟ - 152

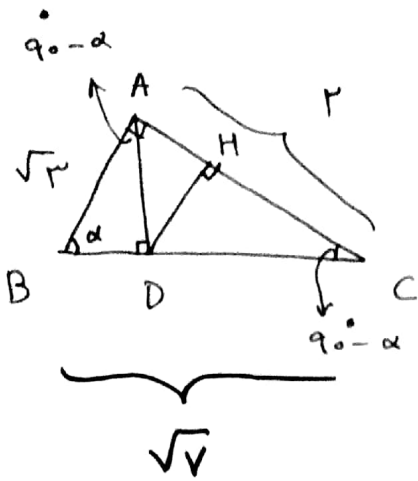


$$\frac{x}{x+10} = \frac{9}{19} \Rightarrow 19x = 9x + 90 \Rightarrow x = \frac{90}{10} = 9$$

$$\frac{x}{10} = \frac{y}{10} \Rightarrow y = \frac{10}{10} = 1$$

$$\Rightarrow x + y + 10 = \frac{90}{10} + 1 + 10 = 10 + 1 + 10 = 21$$

(3) - 188



$$BC = \sqrt{3^2 + 4^2} = \sqrt{7}$$

$$CD = \frac{AC^2}{BC} = \frac{4}{\sqrt{7}}$$

$$\triangle HCD \sim \triangle ABD \Rightarrow \frac{CD}{AB} = \frac{CH}{AD}$$

$$= \left(\frac{\frac{4}{\sqrt{7}}}{\sqrt{3}} \right)^2 = \frac{16}{21}$$