

دنباله اول: 1, 2, 4, 8, 16

دنباله سوم: 2, 4, 8, 16, 32

۱۰۱ - گزینه ۴

دنباله دوم: 1, 3, 9, 27, 81

دنباله چهارم: 3, 6, 12, 24, 48

دنباله پنجم: 5, 10, 20, 40, 80

دنباله ششم: 4, 8, 16, 32, 64

دنباله هفتم: 6, 12, 24, 48, 96

هفت دنباله با شرایط خواسته شده می توان ساخت

۱۰۲ - گزینه ۱  $y = \frac{-\Delta'}{\alpha} = 2 \rightarrow \frac{-(36 - m(5m - 1))}{m} = 2 \rightarrow 5m^2 - 3m - 36 = 0$

$\rightarrow m = \frac{-b \pm \sqrt{\Delta}}{2\alpha} = \frac{3 \pm \sqrt{729}}{10} = \frac{3 \pm 27}{10}$   
 $\rightarrow m = 3$  OK (مینه)  $\rightarrow \gamma = \frac{-b}{2\alpha} = \frac{12}{2m} = \frac{12}{6} = 2$   
 عدد  $m = -2/4$

۱۰۶ - گزینه ۱  $x^2 + 6x + \alpha = 0$   
 $S = \alpha + \beta = -\frac{b}{\alpha} = -6$   
 $P = \alpha\beta = \frac{c}{\alpha} = \alpha$   
 $d = |\alpha - \beta| = \frac{\sqrt{\Delta}}{|\alpha|} = \sqrt{36 - 4\alpha}$

چون  $\alpha < \beta$  است پس  $\alpha - \beta$  منفی است پس  $\alpha - \beta = -\sqrt{36 - 4\alpha}$

$3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85 \rightarrow \frac{5}{2}\alpha^2 + \frac{1}{2}\alpha^2 + \frac{5}{2}\beta^2 - \frac{1}{2}\beta^2 = \frac{5}{2}(\alpha^2 + \beta^2) + \frac{1}{2}(\alpha^2 - \beta^2) =$

$\frac{5}{2}(S^2 - 2P) + \frac{1}{2}(\alpha + \beta)(\alpha - \beta) = \frac{5}{2}(36 - 2\alpha) + \frac{1}{2}(-6)(-\sqrt{16(36 - 4\alpha)}) =$

$90 - 5\alpha + 12\sqrt{\frac{9 - \alpha}{4}} = 85 + 12\sqrt{2} \rightarrow \begin{cases} 90 - 5\alpha = 85 \rightarrow \alpha = 1 \\ \frac{9 - \alpha}{4} = 2 \rightarrow \alpha = 1 \end{cases}$

۱۰۷ - گزینه ۳  $\alpha^3 = t \rightarrow \frac{1}{t+1} + \frac{1}{t-1} = 2 \rightarrow t-1+t+1 = 2(t^2-1) \rightarrow t^2 = t+1$

$\left(\frac{1}{t-\sqrt{t+1}} + \frac{1}{t+\sqrt{t+1}}\right)^{1401} = \left(\frac{t+\sqrt{t+1}+t-\sqrt{t+1}}{(t+1-\sqrt{t+1})(t+1+\sqrt{t+1})}\right)^{1401} = \left(\frac{2t+2}{(t+1)^2-t}\right)^{1401}$

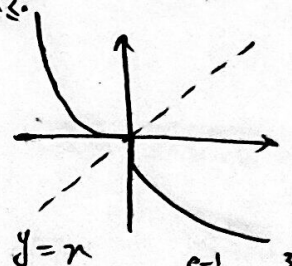
$= \left(\frac{2(t+1)}{t^2+t+1}\right)^{1401} \xrightarrow{t^2=t+1} \left(\frac{2(t+1)}{2(t+1)}\right)^{1401} = (1)^{1401} = 1401$

$f(x) = x^2\sqrt{x^2} = x^2|x| = \begin{cases} x^3 & x > 0 \rightarrow \text{البدایه صعودی} \\ -x^3 & x \leq 0 \rightarrow \text{البدایه نزولی} \end{cases}$

$-x^3 = y \rightarrow x = -\sqrt[3]{y}$  جای  $x$  در  $y$   $y = f^{-1}(y) = -\sqrt[3]{y}$

$x \leq 0 \rightarrow -x \geq 0 \rightarrow -x^3 \geq 0 \rightarrow y \geq 0 \rightarrow R_f = D_{f^{-1}} : x \geq 0$

۱۰۸ - گزینه ۴  $f(x) = x^3$



$f^{-1}(x) = -\sqrt[3]{x}$   
 $x > 0$

$A(x, y) \in y = a - x$  ,  $B(-3, 2)$ ,  $C(-1, 4)$

۱۰۹ - گزینه ۱

$$\left. \begin{aligned} AB &= \sqrt{(x+3)^2 + (y-2)^2} = \sqrt{29} \rightarrow x^2 + y^2 + 6x - 4y - 16 = 0 \\ AC &= \sqrt{(x+1)^2 + (y-4)^2} = 5 \rightarrow x^2 + y^2 + 2x - 8y - 8 = 0 \end{aligned} \right\} \begin{aligned} \text{کم} \\ \text{کنیم} \end{aligned} \rightarrow 4x + 4y - 8 = 0$$

$y = a - x \rightarrow 4x + 4(a - x) - 8 = 0 \rightarrow 4a = 8 \rightarrow a = 2$

۱۱۰ - گزینه ۱

$f(x) = \frac{\sqrt{2}x}{3x - \sqrt{2}} \rightarrow f(\sqrt{2}) = \frac{\sqrt{2}(\sqrt{2})}{3\sqrt{2} - \sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$f(f(\sqrt{2})) = f(\frac{1}{\sqrt{2}}) = \frac{\sqrt{2}(\frac{1}{\sqrt{2}})}{3(\frac{1}{\sqrt{2}}) - \sqrt{2}} = \frac{1}{\frac{1}{2}\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \rightarrow f(f(f(\sqrt{2}))) = f(\sqrt{2}) = \frac{1}{\sqrt{2}}$

$5^x = 10 \rightarrow x = \log_5 10 = \log_5 2 \times 5 = \log_5 2 + \log_5 5 \rightarrow x = \log_5 2 + 1 \rightarrow \log_5^2 = x - 1$

۱۱۱ - گزینه ۳

$2^{f(x)} = 20 \rightarrow f(x) = \log_2 20 = \frac{\log_2 20}{\log_2 5} = \frac{\log_2 2 + \log_2 10}{\log_2 5} = \frac{x-1 + x}{x-1} = \frac{2x-1}{x-1}$

$\left. \begin{aligned} A &= B + 45^\circ \\ A + B + C &= 180^\circ \end{aligned} \right\} \text{معمولی} \rightarrow A = 90^\circ \rightarrow B = 45^\circ, C = 45^\circ$

۱۱۲ - گزینه ۲

$2 \cos A \sin B - \sin C = 2 \cos 90^\circ \sin 45^\circ - \sin 45^\circ = 0 - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$

$f(x) = a \cos(bx + c)$  ,  $b > 0$  ,  $0 < c < \pi$

۱۱۳ - گزینه ۱

$y_{max} = |a| = \frac{1}{4} \rightarrow a = \pm \frac{1}{4}$

$\left. \begin{aligned} f(\frac{1}{4}) = 0 &\rightarrow \pm \frac{1}{4} \cos(\frac{1}{4}b + c) = 0 \rightarrow \frac{1}{4}b + c = \frac{\pi}{2} \\ f(\frac{5}{4}) = 0 &\rightarrow \pm \frac{1}{4} \cos(\frac{5}{4}b + c) = 0 \rightarrow \frac{5}{4}b + c = \frac{3\pi}{2} \end{aligned} \right\} \rightarrow b = \pi, c = \frac{\pi}{4}$

$f(x) = \pm \frac{1}{4} \cos(\pi x + \frac{\pi}{4}) \xrightarrow{\text{با توجه به شکل}} f(0) > 0 \rightarrow f(x) = \frac{1}{4} \cos(\pi x + \frac{\pi}{4}) \rightarrow a = \frac{1}{4}$

$\sin x + \tan 60^\circ \cos x = \sqrt{2} \rightarrow \sin x + \frac{\sin 60^\circ}{\cos 60^\circ} \cos x = \sqrt{2} \rightarrow$

۱۱۴ - گزینه ۳

$\frac{\sin x \cos 60^\circ + \sin 60^\circ \cos x}{\frac{1}{2}} = \sqrt{2} \rightarrow \sin(x + \frac{\pi}{3}) = \frac{\sqrt{2}}{2} \rightarrow \sin(x + \frac{\pi}{3}) = \sin \frac{\pi}{4}$

$\left. \begin{aligned} x + \frac{\pi}{3} &= 2k\pi + \pi - \frac{\pi}{4} \rightarrow x = 2k\pi + \frac{5\pi}{12} = \{ \frac{5\pi}{12} \} \\ x + \frac{\pi}{3} &= 2k\pi + \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12} = \{ -\frac{\pi}{12}, \frac{23\pi}{12} \} \end{aligned} \right\} \rightarrow \text{مجموع جوابها} = \frac{9\pi}{4}$

115 - گزینه 4

$$\lim_{x \rightarrow -1} \frac{\sqrt{2x+3} - \sqrt{3x+4}}{1 + \sqrt[3]{x}} = \frac{0}{0} \xrightarrow{HOP} \lim_{x \rightarrow -1} \frac{\frac{2}{2\sqrt{2x+3}} - \frac{3}{2\sqrt{3x+4}}}{\frac{1}{3\sqrt[3]{x^2}}}$$

$$= \frac{\frac{2}{2} - \frac{3}{2}}{\frac{1}{3}} = \frac{-\frac{1}{2}}{\frac{1}{3}} = \frac{-3}{2}$$

116 - گزینه 3

$|x^3| = x^2 \rightarrow |x^2 \cdot x| = x^2 \rightarrow x^2|x| - x^2 = 0 \rightarrow x^2(|x|-1) = 0 \rightarrow x = 0, \pm 1$

ضابطه  $f(x) = [x^2] - [x]$  در بازه  $(-1, 1)$  یا  $(1, \infty)$  یا  $(-\infty, -1)$

دامنه به سه نقطه تقسیم می شود

$$f(x) = \begin{cases} |x| + [-x] & -1 < x < 1, x \neq 0 \\ 1 + \cos \pi x & x = \pm 1, 0 \\ [x^2] - [x] & x < -1 \text{ یا } x > 1 \end{cases}$$

117 - گزینه 4

$n=1 \rightarrow f(x) = x^4 + 2x^3 + x^6 + 3x^5 + 16a$

$x+2=0 \rightarrow x=-2 \rightarrow f(-2) = 0 \rightarrow 16 - 16 + 64 - 96 + 16a = 0 \rightarrow a=2$

$f(x) = x^6 + 3x^5 + x^4 + 2x^3 + 32$

$x^2 + 2x - 3 = 0 \rightarrow \begin{cases} x=1 \\ x=-3 \end{cases} \rightarrow f(1) = 39$

کنترل نزدیکها  $R(x) = -5x + 44$

118 - گزینه 3

مقدار جمله آخر در دنباله  $= 1+2+3 = 6$

مقدار جمله آخر در دنباله چهارم  $= 1+2+3+6 = 12$

مقدار جمله آخر در دنباله پنجم  $= 1+2+3+6+12 = 24$

...

مقدار جمله آخر در دنباله دوازدهم  $= 1+2+3+6+12+\dots+a_{10} = 1+2 + \frac{3(1-(2)^{10})}{1-2} = 3072$

مجموعه های  $(3073, 3074, \dots, 6144)$   $\xrightarrow[\text{صافی}]{\text{دنباله}}$   $\bar{M} = \frac{6144 + 3073}{2} = 4608.5$

119 - گزینه 1

$f(x) = \frac{|ax+1| + 2x}{|x|+b}$

$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{ax+2x}{x} = a+2$

$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{-ax+2x}{-x} = a-2$

اول حالت 1  $\begin{cases} a+2 = -b \\ a-2 = b \end{cases} \rightarrow a=0, b=-2$

دوم حالت 2  $\begin{cases} a+2 = b \\ a-2 = -b \end{cases} \rightarrow a=0, b=2$

$|x|+b=0 \rightarrow \begin{cases} x \geq 0 \rightarrow x+b=0 \rightarrow x=-b \\ x < 0 \rightarrow -x+b=0 \rightarrow x=b \end{cases}$

$a=0, b=-2 \rightarrow f(x) = \frac{1+2x}{|x|-2} \rightarrow \lim_{x \rightarrow 1^+} \frac{1+2x}{|x|-2} = \frac{3}{-1} = -3$

120 - گزینه 2  
 $f(x) = g(x) \rightarrow \sin x + \frac{1}{2} \cos x = \frac{3}{2} \sin x \rightarrow \frac{1}{2} \cos x = \frac{1}{2} \sin x$   
 $\div \cos x \rightarrow \tan x = 1 \rightarrow x = k\pi + \frac{\pi}{4} = \left\{ \frac{\pi}{4} \right\} \in [0, \pi] \rightarrow y = 3\frac{\sqrt{2}}{4} \rightarrow A\left(\frac{\pi}{4}, 3\frac{\sqrt{2}}{4}\right)$

$f'(x) = \cos x - \frac{1}{2} \sin x \rightarrow f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{4} = m$

$y - 3\frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4} \left(x - \frac{\pi}{4}\right) \xrightarrow{y=0} -3\frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4} \left(x - \frac{\pi}{4}\right) \rightarrow x = \frac{\pi}{4} - 3$

121 - گزینه 3  
 $T = 5 \rightarrow f(x+5) = f(x) \rightarrow f'(x+5) = f'(x) \xrightarrow{x=-1} f'(4) = f'(-1)$

$g(x) = f(x+1) + f(3x+10) \rightarrow g'(x) = f'(x+1) + 3f'(3x+10) \xrightarrow{x=-2}$

$g'(-2) = f'(-1) + 3 \underbrace{f'(4)}_{f'(-1)} \rightarrow g'(-2) = 4f'(-1) = 4 \times \frac{3}{2} = 6$

122 - گزینه 3  
 $f(x) = (x-4)\sqrt[3]{x+3} \rightarrow f(5) = 2$

$f'(x) = \sqrt[3]{x+3} + \frac{1}{3\sqrt[3]{(x+3)^2}} (x-4) \rightarrow f'(5) = 2 + \frac{1}{12} (1) = \frac{25}{12}$

$\lim_{h \rightarrow 0} \frac{f^2(5-h) - 3f(5-h) + 2}{5h - h^2} = \frac{0}{0} \xrightarrow{\text{لانه } h \rightarrow 0} \lim_{h \rightarrow 0} \frac{-2f(5-h)f'(5-h) + 3f'(5-h)}{5 - 2h}$   
 $= \frac{-2f(5)f'(5) + 3f'(5)}{5} = \frac{-2(2) \left(\frac{25}{12}\right) + 3\left(\frac{25}{12}\right)}{5} = \frac{-\frac{25}{2} + \frac{75}{12}}{5} = \frac{-\frac{25}{12}}{5} = -\frac{5}{12}$

123 - گزینه 1  
 $A(-1, 1), x = -1 < 0 \rightarrow f(x) = -x^3 + 3ax^2 + b$

$f(-1) = 1 \rightarrow 1 + 3a + b = 1 \rightarrow 3a + b = 0$

$f'(-1) = 0 \rightarrow f'(x) = -3x^2 + 6ax \rightarrow f'(-1) = -3 - 6a = 0 \rightarrow a = -\frac{1}{2}, b = \frac{3}{2}$

124 - گزینه 4  
 $\lim_{x \rightarrow \pm\infty} f(x) = \lim_{x \rightarrow \infty} \frac{ax+3}{(a+1)x+a-1} = \frac{a}{a+1}$ ,  $(a+1)x+a-1=0 \rightarrow x = \frac{1-a}{a+1}$

$w\left(\frac{1-a}{a+1}, \frac{a}{a+1}\right)$ ,  $f'(x) = 3x+1 \rightarrow f'\left(\frac{1-a}{a+1}\right) = 3\left(\frac{1-a}{a+1}\right) + 1 = 0 \rightarrow a = 2$   
 $f(x) = \frac{2x+3}{3x+1} \xrightarrow{y=0} 2x+3=0 \rightarrow x = -\frac{3}{2}$