

۱۰۱- گزینه ۲

$$\sqrt[4]{\frac{1}{4+\sqrt{7}}} \sqrt[4]{(1+\sqrt{7})^2} = \sqrt[4]{\frac{1+7+2\sqrt{7}}{4+\sqrt{7}}} = \sqrt[4]{\frac{2(4+\sqrt{7})}{4+\sqrt{7}}} = \sqrt[4]{2}$$

۱۰۲- گزینه ۴

$$a_n = an + b \rightarrow \begin{cases} a_5 = 5a + b = 8 \\ a_{10} = 10a + b = 5 \end{cases} \rightarrow a = -\frac{3}{5}, b = 11$$

$$a_n = -\frac{3}{5}n + 11 \rightarrow a_{16} = -\frac{48}{5} + 11 = \frac{7}{5} = 1.4$$

۱۰۳- گزینه ۱

$$y = ax^2 + (3+2a)x \xrightarrow{\text{نقطهٔ مبدا}} \rightarrow a > 0, f(0) = 0 = c \checkmark$$



$$S = x_1 + x_2 = -\frac{b}{a} \geq 0 \rightarrow -\frac{(3+2a)}{a} \geq 0 \xrightarrow{a > 0} 3+2a \leq 0 \rightarrow a \leq -\frac{3}{2}$$

$$P = x_1 x_2 = \frac{c}{a} = 0 \checkmark$$

انتزاک بین  $a > 0$  و  $a \leq -\frac{3}{2}$  نمی‌آید.

۱۰۴- گزینه ۳

$$\frac{4-2x}{3x+1} \geq 0 \xrightarrow{\text{تعیین علامت}} \frac{x}{3x+1} \left| -\frac{1}{3} \quad \frac{2}{1} \right| \rightarrow -\frac{1}{3} < x \leq \frac{2}{1} \xrightarrow{\times 3} -1 < 3x \leq 6$$

$$-1 < 3x \leq 6 \rightarrow [3x] = -1, 0, 1, 2, 3, 4, 5, 6$$

۱۰۵- گزینه ۳

$$\begin{cases} f(x) = b - 3ax \xrightarrow{\text{نقطهٔ مبدا}} -3a = 0 \rightarrow a = 0 \rightarrow f(x) = b \\ g(x) = c - (3b-3)x \xrightarrow{\text{نقطهٔ مبدا}} 3b-3 = 0 \rightarrow b = 1 \rightarrow g(x) = c \end{cases} \rightarrow f(x) = 1$$

$$f(x) + g(x) = 5 \rightarrow 1 + c = 5 \rightarrow c = 4 \rightarrow g(x) = 4 \rightarrow b \times c = 1 \times 4 = 4$$

۱۰۶- گزینه ۴

$$f(x) = 4x - x^2 \xrightarrow{\text{دو واحد به چپ}} y = 4(x+2) - (x+2)^2 = -x^2 + 4$$

$$\left. \begin{aligned} f(x) &= 4x - x^2 \\ y &= -x^2 + 4 \end{aligned} \right\} \rightarrow -x^2 + 4 = 4x - x^2$$

$$\rightarrow x = 1 \rightarrow y = 3 \rightarrow A(1, 3), O(0, 0) \rightarrow OA = \sqrt{10}$$

۱۰۷- گزینه ۳

$$3x^2 - ax + 4 = 0 \rightarrow S = \alpha + \beta = -\frac{b}{a} = \frac{a}{3}, P = \frac{c}{a} = \alpha\beta = \frac{4}{3}$$

$$\alpha = 3\beta \rightarrow 3\beta(\beta) = \frac{4}{3} \rightarrow \beta^2 = \frac{4}{9} \rightarrow \beta = \pm \frac{2}{3}, \alpha = \pm 2 \rightarrow$$

$$S = \alpha + \beta = \frac{a}{3} \rightarrow \begin{cases} \frac{2}{3} + 2 = \frac{8}{3} = \frac{a}{3} \rightarrow a = 8 \\ -\frac{2}{3} - 2 = -\frac{8}{3} = \frac{a}{3} \rightarrow a = -8 \end{cases} \rightarrow \text{مجموعه } \alpha = 16$$

۱۰۸- گزینه ۲

$$\frac{\sqrt{x+1}(\sqrt{x-1}-3+\sqrt{x-1}+3)}{(\sqrt{x-1}+3)(\sqrt{x-1}-3)} = \frac{x-1}{\sqrt{x-1}} \rightarrow \frac{2\sqrt{x+1}\sqrt{x-1}}{x-1-9} = \frac{\sqrt{x+1}\sqrt{x-1}}{\sqrt{x-1}}$$

$$x > 1 \rightarrow \frac{2\sqrt{x+1}}{x-10} = 1 \rightarrow x-10 = 2\sqrt{x+1} \xrightarrow{\text{باز کردن}} x^2 - 24x + 96 = 0 \rightarrow x = \frac{12 \pm 4\sqrt{3}}{2} \rightarrow \begin{cases} x = 4(3+\sqrt{3}) \\ x = 4(3-\sqrt{3}) \end{cases}$$

۱۰۹- گزینه ۲  $y = x^3 - x + 1 \xrightarrow[\text{گزینه ۲}]{\text{کنترل گزینہ}} y = \frac{1}{8} - \frac{1}{2} + 1 = \frac{1-4+8}{8} = \frac{5}{8} \rightarrow A(\frac{1}{2}, \frac{5}{8})$

۱۱۰- گزینه ۴  $g(f(x)) = 5x^2 + 11 \rightarrow g(2x) = 5x^2 + 11 \rightarrow g(2 \times \frac{x}{2}) = 5(\frac{x}{2})^2 + 11$   
 $\rightarrow g(x) = \frac{5x^2}{4} + 11 \rightarrow g(x-7) = \frac{5}{4}(x-7)^2 + 11 \rightarrow \min(7, 11) \rightarrow y_{\min} = 11$

۱۱۱- گزینه ۱  $f(x) = (-9 + k^2)x^3 + 5 \xrightarrow{\text{اکتفا بر دایره}} (-9 + k^2) < 0 \rightarrow k^2 < 9 \rightarrow$   
 $-3 < k < 3 \rightarrow k = -2, -1, 0, 1, 2 \rightarrow k \text{ مجموع متناظر صغی} = 0$

۱۱۲- گزینه ۱  $\tan(\frac{\pi}{4} - x) = \frac{1-m}{2+m} \rightarrow \tan \alpha = \frac{1-m}{2+m}$   
 $-\frac{\pi}{4} < x < \frac{\pi}{4} \rightarrow \frac{\pi}{4} > -x > -\frac{\pi}{4} \rightarrow \frac{\pi}{2} > \frac{\pi}{4} - x > 0 \rightarrow 0 < \alpha < \frac{\pi}{2} \rightarrow$   
 $\tan \alpha > 0 \rightarrow \frac{1-m}{2+m} > 0 \xrightarrow{\text{تقسیم عددها}} \frac{1-m}{2+m} > 0 \xrightarrow{\text{بالاتر صغی}} \frac{1-m}{2+m} > 0 \rightarrow -2 < m < 1$

۱۱۳- گزینه ۳  $2\sin^2 x + \cos^2 x = \frac{4}{3} \xrightarrow{\div \cos^2 x} 2\tan^2 x + 1 = \frac{4}{3}(\frac{1}{\cos^2 x}) \rightarrow$   
 $2\tan^2 x + 1 = \frac{4}{3}(1 + \tan^2 x) \rightarrow 6\tan^2 x + 3 = 4 + 4\tan^2 x \rightarrow 2\tan^2 x = 1 \rightarrow \tan^2 x = \frac{1}{2}$

۱۱۴- گزینه ۳  $y = a \cos bx + c \rightarrow T = \frac{2\pi}{|b|} = 4\pi \rightarrow |b| = \frac{1}{2} \rightarrow b = \pm \frac{1}{2}$   
 $c = \frac{y_{\max} + y_{\min}}{2} = \frac{5+1}{2} = 3, |a| = \frac{y_{\max} - y_{\min}}{2} = \frac{5-1}{2} = 2 \rightarrow a = \pm 2$   
 $y = \pm 2 \cos(\pm \frac{1}{2}x) + 3 \xrightarrow{\text{بالاتر صغی}} y = 2 \cos(\frac{1}{2}x) + 3 \rightarrow a=2, b=\pm \frac{1}{2}, c=3$

۱۱۵- گزینه ۴  $8 \cos x = 1 + \tan^2 x \rightarrow 8 \cos x = \frac{1}{\cos^2 x} \rightarrow 8 \cos^3 x = 1 \rightarrow \cos x = \frac{1}{2}$   
 $\cos x = \cos \frac{\pi}{3} \rightarrow x = 2k\pi \pm \frac{\pi}{3} = \{ \frac{\pi}{3}, 2\pi - \frac{\pi}{3} \}$

۱۱۶- گزینه ۱  $\log_8 18 = m \rightarrow \frac{\log_2 18}{\log_2 8} = m \rightarrow \frac{\log_2 2 \times 3^2}{\log_2 2^3} = m \rightarrow \frac{1+2\log_2 3}{3} = m \rightarrow 2\log_2 3 = 3m-1$   
 $\rightarrow \log_2 3 = \frac{3m-1}{2}, \log_4 12 = \log_4 4 \times 3 = \log_4 4 + \log_4 3 = 1 + \frac{1}{2}\log_2 3 = 1 + \frac{1}{2}(\frac{3m-1}{2}) = \frac{3m+3}{4}$



117 - گزینه 3

$$\left. \begin{aligned} f(x) &= a + b\left(\frac{1}{2}\right)^x \xrightarrow{(0,0)} a + b\left(\frac{1}{2}\right)^0 = 0 \rightarrow a + b = 0 \\ f^{-1}(-1) &= -1 \rightarrow f(-1) = -1 \rightarrow a + b\left(\frac{1}{2}\right)^{-1} = -1 \rightarrow a + 2b = -1 \end{aligned} \right\} \rightarrow \begin{aligned} a &= 1, b = -1 \\ a - b &= 2 \end{aligned}$$

118 - گزینه 4

$$S = \frac{(x_1 - \bar{x})^2 + \dots + (x_8 - \bar{x})^2 + (x_9 - \bar{x})^2}{9} = \frac{8(1) + 0}{9} = \frac{8}{9} \rightarrow S = \frac{2\sqrt{2}}{3}$$

119 - گزینه 1

$$1, 2, 3, \dots, n \quad \bar{x} = \tilde{x} \rightarrow (\bar{x} + 2) - (\tilde{x} + 2) = 0$$

چون داده‌ها دنباله حسابی هستند میانگین و میانگین برابرند.

120 - گزینه 2

$$\lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x^3 - [x^3]} = \lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x^3 - 8} = \frac{0}{0} \xrightarrow{\text{Hop}} \lim_{x \rightarrow 2} \frac{2x}{3x^2} = \frac{1}{3}$$

121 - گزینه 3

$$\lim_{x \rightarrow 1^+} (4 - [x])g(x) = 6 \rightarrow 3 \lim_{x \rightarrow 1^+} g(x) = 6 \rightarrow \lim_{x \rightarrow 1^+} g(x) = 2$$

$$\lim_{x \rightarrow 1^+} \frac{\sqrt{ax^2 + bx + c}}{|x - 1|} = \lim_{x \rightarrow 1^+} \frac{\sqrt{ax^2 + bx + c}}{x - 1} = 2 \rightarrow \sqrt{ax^2 + bx + c} = \sqrt{4(x - 1)^2} = \sqrt{4x^2 - 4x + 4}$$

$$\lim_{x \rightarrow +\infty} g(x) = \lim_{x \rightarrow +\infty} \frac{\sqrt{4x^2 - 4x + 4}}{|x - 1|} = \frac{\infty}{\infty} \xrightarrow{\text{Hop}} \lim_{x \rightarrow +\infty} \frac{12x}{x} = 2$$

122 - گزینه 1

$$\lim_{x \rightarrow 0} \frac{f(x)}{x} = \lim_{x \rightarrow 0} \frac{x \left( \sqrt{\frac{2x+1}{5x+9}} \right)^3}{x} = \lim_{x \rightarrow 0} \left( \sqrt{\frac{2x+1}{5x+9}} \right)^3 = \left( \sqrt{\frac{1}{9}} \right)^3 = \left( \frac{1}{3} \right)^3 = \frac{1}{27}$$

123 - گزینه 4

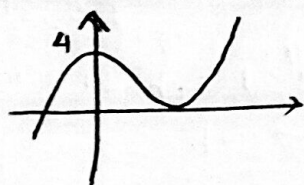
$$4y - 3x = n \rightarrow y = \frac{3}{4}x + \frac{n}{4} \rightarrow m_{\text{مماس}} = f'(1) = \frac{3}{4}$$

$$f(x) = \frac{x^2 + mx + 1}{x + 3} \rightarrow f'(x) = \frac{(2x + m)(x + 3) - (x^2 + mx + 1)}{(x + 3)^2} \rightarrow f'(1) = \frac{8 + 4m - 2 - m}{16}$$

$$\frac{6 + 3m}{16} = \frac{3}{4} \rightarrow \boxed{m = 2} \rightarrow f(x) = \frac{x^2 + 2x + 1}{x + 3} \xrightarrow{x=1} f(1) = 1 \rightarrow A(1, 1)$$

$$4y - 3x = n \xrightarrow{A(1,1)} 4 - 3 = n \rightarrow \boxed{n = 1}$$

$$m + n = 2 + 1 = 3$$



$$f(x) = x^3 + ax^2 + bx + c \xrightarrow{(0,4)} c = 4$$

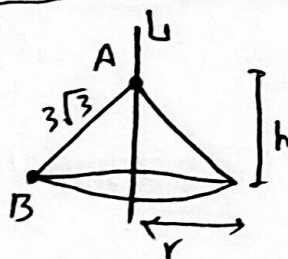
۱۲۴ گزینه ۲

$$f'(0) = 0 \rightarrow f'(x) = 3x^2 + 2ax + b \rightarrow f'(0) = b = 0$$

$$f(x) = x^3 + ax^2 + 4 \rightarrow f'(x) = 3x^2 + 2ax = 0 \rightarrow x(3x + 2a) = 0 \rightarrow \begin{cases} x = 0 \\ x = -\frac{2a}{3} \end{cases}$$

$$\min(-\frac{2a}{3}, 0) \in y = f(x) \rightarrow (-\frac{2a}{3})^3 + a(-\frac{2a}{3})^2 + 4 = 0 \rightarrow -\frac{8a^3}{27} + \frac{4a^3}{9} = -4$$

$$\frac{4a^3}{27} = -4 \rightarrow a^3 = -27 \rightarrow a = -3 \rightarrow x_{\min} = \frac{-2a}{3} \xrightarrow{a=-3} x_{\min} = 2$$



$$V = \frac{1}{3} \pi r^2 h$$

۱۲۵ گزینه ۲

$$r^2 + h^2 = 27 \rightarrow r^2 = 27 - h^2$$

$$V = \frac{1}{3} \pi (27 - h^2) h = \frac{\pi}{3} (27h - h^3)$$

$$V'_h = \frac{\pi}{3} (27 - 3h^2) = 0 \rightarrow h^2 = 9 \rightarrow h = 3$$

RFZ 1 2 3 4

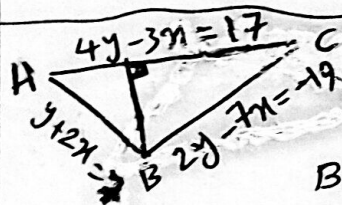
۱۲۶ - گزینه ۱

$$R, Z, X \rightarrow \binom{4}{2} = 6, F, X, X \rightarrow \binom{4}{3} = 4 \rightarrow 4 + 6 = 10$$

$$X, X \rightarrow \binom{5}{4} = 5$$

$$P(A) = 0,08, P(B) = 0,5 \rightarrow P(A \cap B) = P(A) \cdot P(B) = 0,08 (0,5) = 4\%$$

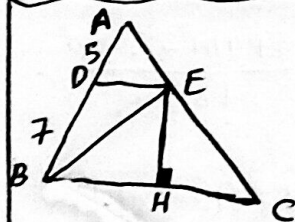
۱۲۷ - گزینه ۴



$$\begin{cases} 2y - 7x = -19 \\ y + 2x = 7 \end{cases} \xrightarrow{\text{از اول}} \begin{cases} 2y - 7x = -19 \\ 2y + 4x = 14 \end{cases} \xrightarrow{\text{تفاوت}} \begin{cases} 2y - 7x = -19 \\ 2y + 4x = 14 \end{cases} \rightarrow \begin{cases} -11x = -33 \\ x = 3 \end{cases} \rightarrow y = 13 \rightarrow B(3, 13)$$

$$BH = \frac{|14(1) - 3(3) - 17|}{\sqrt{16 + 9}} = \frac{22}{5} = 4,4$$

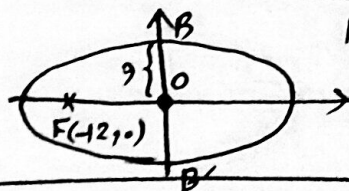
۱۲۸ - گزینه ۱



چون DE موازی BC است پس BC, DE, EH  
مکعب است، (ارتفاع) و (مساحت) BDE و BCE است پس:

۱۲۹ - گزینه ۴

$$\frac{S_{BCE}}{S_{BDE}} = \frac{\frac{1}{2} BC \times EH}{\frac{1}{2} DE \times EH} = \frac{BC}{DE} = \frac{AB}{AD} = \frac{12}{5} = 2,4$$



$$BB' = 2b = 18 \rightarrow b = 9, OF' = c = 12$$

۱۳۰ - گزینه ۲

$$a^2 = b^2 + c^2 = 144 + 81 = 225 \rightarrow c = 15 \rightarrow e = \frac{c}{a} = \frac{12}{15} = 0,8$$