۱- تست ۲

$$S_9 = \frac{a(1-q^9)}{1-q} = 17 S_3$$

$$\frac{a(1-q^9)}{1-q} = 17 \frac{a(1-q^3)}{1-q}$$

$$\frac{1-q^9}{1-q^3} = 17 \Longrightarrow \frac{(1-q^3)(1+q^3+q^6)}{1-q^3} = 17$$

$$q^3(1+q^3) = \underbrace{17}_{16+1} \Longrightarrow q=2$$

$$\frac{a_3}{a} = \frac{aq^2}{a} = 4$$

۲- تست ۴

$$x^2 + \frac{10}{x^2+1} = 9 \Rightarrow x^2+1 + \frac{10}{x^2+1} = 10$$

$$x^2+1 = t \Rightarrow t + \frac{10}{t} = 10$$

$$(x^2+1)^2 + \frac{100}{(x^2+1)^2} = t^2 + \frac{100}{t^2} = t^2 + \left(\frac{10}{t}\right)^2$$

$$t^2 + \left(\frac{10}{t}\right)^2 = \left(t + \frac{10}{t}\right)^2 - 20 = 10^2 - 20 = 80$$

$$\frac{4\alpha+\beta^5}{5\beta^2} = \frac{4}{5}\frac{\alpha}{\beta^2} + \frac{1}{5}\beta^3$$

$P = \alpha\cdot\beta = 2$

$S = \alpha+\beta = 5$

- 5

$$A = \frac{4}{5}\frac{\alpha}{\beta^2} + \frac{1}{8}\beta^3$$

$$A = \frac{4}{5}\frac{\beta}{\alpha^2} + \frac{1}{5}\alpha^3$$

$$\Rightarrow 2A = \frac{4}{5}\frac{\alpha^3+\beta^3}{\alpha^2\beta^2} + \frac{1}{5}(\alpha^3+\beta^3)$$

$$2A = \frac{4}{5}(\alpha^3+\beta^3) = \frac{4}{5}(S^3 - 3SP) = \frac{4}{5}(125-30)$$

$$A = \frac{1}{5}(95) = 19$$

- 7

$$f'(x) = \frac{-5\times\frac{3mx-8}{2\sqrt{mx^2-8x+29}}}{mx^2+8x+29} = 0$$

$$3mx - 8 = 0 \Rightarrow x = \frac{8}{3m} = \frac{4}{m}$$

$$f\left(\frac{4}{m}\right) = \frac{5}{\sqrt{m\left(\frac{16}{m^2}\right) - \frac{32}{m} + 29}} = 1$$

$$25 = \frac{-16}{m} + 29 \Rightarrow \frac{16}{m} = 16 \Rightarrow m = \frac{16}{16}$$

$$[m] = 1$$

۸- گزینه (۱)

$p(x) = x^5 + ax^2$

$q(x) = x^4 + 2x^3 + 2x^2$

$x^n + x^2$

$na = ?$

$q(x) = x^2(x^2 + 2x + 2)$

$p(x) = x^2(x^3 + a)$

$q(x) = x^2(x+1)(x+2)$

$p(x) = x^2(x^3 + a)$

$= x^3(x+1)$

$= x^3 + x^2$

$a = -1$

$x^2(x-1)(x+1)$

$n = 2$

$a = -1$

$n \cdot a = -2$

۹- گزینه (۴)

اگر $x \leq 2 \Rightarrow f(x) = 0 \Rightarrow x = 1$

$f(x) = +1 \Rightarrow$

$x-1 = 1 \Rightarrow x = 2$ ✓

$x+1 = 1 \Rightarrow x = 0$ ✓

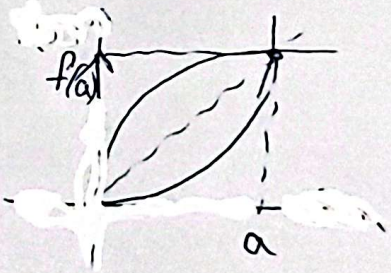
اگر $x > 2 \Rightarrow f(x) = 0 \Rightarrow x+1 = 0 \Rightarrow x = -1$

$x-1 = -1 \Rightarrow x = 0$

$x+1 = -1 \Rightarrow x = -2$ ✓

۱۰- گزینه (۱)

$\lim_{x \to a} \frac{f(x)}{x} = \frac{f(a)}{a}$



$a = f(a) \Rightarrow \lim_{x \to a} \frac{f(x)}{x} = 1$

۱۱- گزینه (۲)

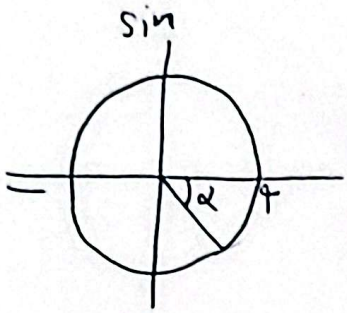
$\log(2x+1) = (\log 5)^2 - (\log 2)^2$

$\log(2x+1) = (\log 5 - \log 2) \times 1 = \log \frac{5}{2}$

$2x+1 = \frac{5}{2} \Rightarrow x = \frac{1}{4} \Longrightarrow \log_{\sqrt{2}}^{x} = \log_{2^{\frac{1}{2}}}^{\frac{1}{4}} = \frac{-1}{\frac{1}{2}} = -2$ (۲)

۱۲- گزینه (۴)

$$\frac{\cos\alpha}{\frac{\sin\alpha}{\sin\alpha}} > 0 \Rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \Rightarrow \cos\alpha > 0$$



$$2\sin\alpha < \sin 2\alpha$$

مثلاً $2\sin(-45^\circ) < \sin(2x-45^\circ)$

یا $2x - \frac{\sqrt{2}}{2} < -1$

۱۳- گزینه (۳)

$$\frac{\sin(-270-15)+2\cos(-90-15)}{2\sin(180-15)+3\sin(360+15)}$$

$$= \frac{\cos 15 - 2\sin 15}{2\sin 15 + 2\sin 15} = \frac{1}{5}\cot 15 - \frac{2}{5} = 0.2\sqrt{2}$$

$$\cot 30 = \frac{\cot^2 15 - 1}{2\cot 15} = \sqrt{3}$$

$$\cot^2 15 - 1 = 2\sqrt{3}\cot 15 \Rightarrow \cot^2 15 - 2\sqrt{3}\cot 15 - 1 = 0$$

$$\cot 15 = \frac{2\sqrt{3} \pm \sqrt{12+4}}{2} = \frac{2\sqrt{3} \pm 4}{2} = \sqrt{3} \pm 2$$

$$\cot 15 > 0 \Rightarrow \cot 15 = \sqrt{3} + 2$$

(۴)

گزینه ۴ - ۱۵

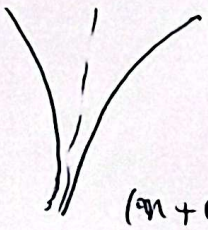
$$\lim_{x\to 0} \frac{k+\cos(\sqrt{a}\,x)}{kx^2} = 2$$

$$H = \lim_{x\to 0} \frac{-\sqrt{a}\sin(\sqrt{a}\,x)}{2kx}$$

$$H = \lim_{x\to 0} \frac{-\sqrt{a}\times\sqrt{a}\cos(\sqrt{a}\,x)}{2k} = \frac{-a}{2k} = 2$$

$$a = -4k \Rightarrow \frac{a}{k} = -4$$

گزینه ۳ - ۱۶



$(x+a)^2$

$$\frac{a^2x^2-2x+1}{(x+a)(mx-2)}$$

mx → -1

$a = 2$

$m = -1$

$$(x+a)(-x-2) = -(x+a)(x+2)$$

$$\lim_{x\to\pi^-} \frac{\cot x}{[x-\pi]} = \frac{-\infty}{-1} = +\infty$$

گزینه ۲ - ۱۷

$an + a = 0 \Rightarrow x = -1$

$$\lim_{x \to -1} \frac{\sqrt{2}\,|x+1|}{|x^2+1|} = \lim_{x \to -1} \frac{\sqrt{2}\,|x+1|}{|x+1||x^2-x+1|}$$

$m = 4$

$$= \frac{\sqrt{2}}{2}$$

$$(-1)^5 + (m-2)(-1) + a^2 = 0 \Rightarrow -1 - m + 2 + a^2 = 0$$

$$a = -1 \Rightarrow -1 - m + 2 + 1 = 0 \Rightarrow m = 2$$

١٩ - تمرين ٢

$$f(x) = \frac{|2x-4|}{\sqrt[3]{x^2}} = \frac{2|x-2|}{\sqrt[3]{x^2}}$$

$$g(x) = \frac{\sqrt{(x-2)^2} + \sqrt[3]{x}}{\sqrt[3]{x^2}} = \frac{|x-2| + \sqrt[3]{x}}{\sqrt[3]{x^2}}$$

$$\left(f(x) - 2g(x)\right)' = \left(\frac{-2\sqrt[3]{x}}{\sqrt[3]{x^2}}\right)'$$

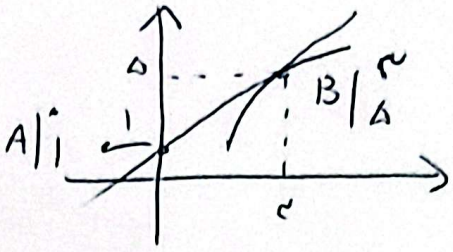
$$= \left(\frac{-2(x)^{\frac{1}{3}}}{x^{\frac{2}{3}}}\right)' = \left(\frac{-2\sqrt[3]{x}}{x^{\frac{2}{3}}}\right)' = \left(-2\sqrt[3]{x} \times x^{-\frac{2}{3}}\right)'$$

$$= -2\sqrt[3]{2}\left(-\frac{2}{3}\right) x^{-\frac{5}{3}} \Rightarrow x = 1 \Rightarrow -2\sqrt[3]{2} \times -\frac{2}{3} \times 1$$

$$= +\frac{2\sqrt[3]{2}}{3}$$

۲۰- گزینه (۴)

$$f(c) = \frac{y_c - y_1}{x_c - x_1} = \frac{5-1}{2-0} = \frac{4}{2}$$



۲۱- گزینه (۱)

$$f(x) = \begin{cases} 2bx + c & x > a \\ 2x^2 & x \le a \end{cases}$$

پیوستگی $\lim_{x \to a} f(x) = f(a) \Rightarrow 2ba + c = 2a^2$

$$f'(x) = \begin{cases} 2b & x > a \\ 4x & x \le a \end{cases}$$

$f'_+(a) = f'_-(a) \Rightarrow 2b = 4a \Rightarrow \boxed{b = 2a}$

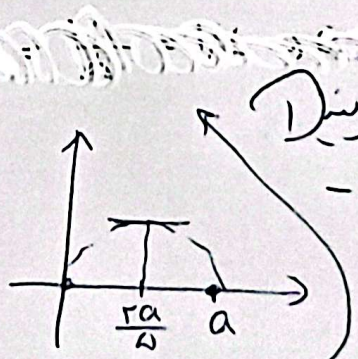
$$a^5 + b - c = a^2 + 2a - (2a^2 - 2ba)$$
$$= a^2 + 2a - (2a^2 - 4a^2) = a^2 + 2a + 2a^2$$
$$= (a+1)^2 - 1$$

۲۲- گزینه (۱)

$$f(x) = \begin{cases} \sqrt[3]{x^2}(x-a) & x \ge a \\ \sqrt[3]{x^2}(a-x) & x < a \end{cases}$$

$$f'(x) = \frac{2}{3\sqrt[3]{x}}(x-a) + \sqrt[3]{x^2}(1) = 0 \quad (V)$$

$x \ge a$



$\Rightarrow 2(x-a) + 3x = 0$

$$f\left(\frac{2}{5}a\right) = \sqrt[3]{\left(\frac{2}{5}a\right)^2} \times \left|\frac{2}{5}a - a\right| = \frac{3}{2}$$

$$\Rightarrow a = \frac{5}{2}$$

هوشمند قصری

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